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Novel Aspects in p-Brane Theories: Weyl-Invariant Light-Like Branes

Eduardo Guendelman and Alexander Kaganovich

Department of Physics, Ben-Gurion University, Beer-Sheva, Israel
email: guendel@bgumail.bgu.ac.il, alexk@bgumail.bgu.ac.il

Emil Nissimov and Svetlana Pacheva

Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia, Bulgaria
email: nissimov@inrne.bas.bg, svetlana@inrne.bas.bg

Abstract

We consider a novel class of Weyl-conformally invariant p-brane theories which describe intrinsically light-like branes for any odd world-volume dimension, hence the acronym *WILL*-branes (Weyl-Invariant Light-Like branes). We discuss in some detail the properties of *WILL*-brane dynamics which significantly differs from ordinary Nambu-Goto brane dynamics. We provide explicit solutions of *WILL*-membrane (i.e., $p = 2$) equations of motion in arbitrary $D = 4$ spherically symmetric static gravitational backgrounds, as well as in product spaces of interest in Kaluza-Klein context. In the first case we find that the *WILL*-membrane materializes the event horizon of the corresponding black hole solutions, thus providing an explicit dynamical realization of the membrane paradigm in black hole physics. In the second "Kaluza-Klein" context we find solutions describing *WILL*-branes wrapped around the internal (compact) dimensions and moving as a whole with the speed of light in the non-compact (space-time) dimensions.

Keywords: Weyl-conformal invariant p-brane actions, light-like p-branes, non-Riemannian volume forms, variable string/brane tension, Kaluza-Klein, event horizons, membrane paradigm.

1 Introduction

The idea of replacing the standard Riemannian integration measure (Riemannian volume-form) with an alternative non-Riemannian volume-form or, more generally, employing on equal footing both Riemannian and non-Riemannian volume-forms to construct new classes of models involving gravity, called *two-measure theories*, has been proposed few years ago [1] and since then it is a subject of active research and developments [2] (for related ideas, see [3]).

Two-measure theories address various basic problems in cosmology and particle physics, and provide plausible solutions for a broad array of issues, such as: scale invariance and its dynamical breakdown; spontaneous generation of dimensionfull fundamental scales; the cosmological constant problem; the problem of fermionic families; applications in modern brane-world scenarios. For a detailed discussion we refer to the series of papers [1, 2].

Subsequently, the idea of employing an alternative non-Riemannian integration measure was applied systematically to string, p-brane and Dp-brane models [4] (for a background on string and brane theories, see refs.[5]). The main feature of these new classes of modified string/brane theories is the appearance of the pertinent string/brane tension as an additional dynamical degree of freedom beyond the usual string/brane physical degrees of freedom, instead of being introduced *ad hoc* as a dimensionfull scale. The dynamical string/brane tension acquires the physical meaning of a world-sheet electric field strength (in the string case) or world-volume $p+1$-form field strength (in the p-brane case) and obeys Maxwell (Yang-Mills) equations of motion or their higher-rank antisymmetric tensor gauge field analogues, respectively. As a result of the latter property the modified-measure string model with dynamical tension yields a simple classical mechanism of "color" charge confinement.

In the next section we proceed to our main task which is the study of a novel class (first proposed in our preceding work [6]) of p-brane theories which are Weyl-conformal invariant for any p and which describe intrinsically light-like branes for any odd $(p+1)$. Thus, their dynamics significantly differs both from the standard Nambu-Goto (or Dirac-Born-Infeld) branes as well as from their modified versions with dynamical string/brane tensions [4] mentioned above.

2 Weyl-Invariant p-Brane Theories

2.1 Standard Nambu-Goto Branes

Let us first briefly recall the standard Polyakov-type formulation of the bosonic p-brane action:

$$S = -\frac{T}{2} \int d^{p+1}\sigma \sqrt{-\gamma} \left[\gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \Lambda(p-1) \right] . \tag{1}$$

Here γ_{ab} is the ordinary Riemannian metric on the $p+1$-dimensional brane world-volume with $\gamma \equiv \det ||\gamma_{ab}||$. The world-volume indices $a, b = 0, 1, \ldots, p$; $G_{\mu\nu}$ denotes the Riemannian metric in the embedding space-time with space-time indices $\mu, \nu = 0, 1, \ldots, D-1$. T is the given *ad hoc* brane tension; the constant Λ can be absorbed by rescaling T (see below Eq.(7)). The equations of motion w.r.t. γ^{ab} and X^μ read:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu} + \gamma_{ab} \frac{\Lambda}{2} (p-1) = 0 , \tag{2}$$

$$\partial_a \left(\sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 , \tag{3}$$

where:

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\kappa} \left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda} \right) \tag{4}$$

is the affine connection for the external metric.

Eqs.(2) when $p \neq 1$ imply:

$$\Lambda \gamma_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} , \tag{5}$$

which in turn allows to rewrite Eq.(2) as:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{p+1} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu} = 0 . \tag{6}$$

Furthermore, using (5) the Polyakov-type brane action (1) becomes on-shell equivalent to the Nambu-Goto-type brane action:

$$S = -T \Lambda^{-\frac{p-1}{2}} \int d^{p+1}\sigma \sqrt{-\det ||\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}||} . \tag{7}$$

2.2 Weyl-Invariant Branes: Action and Equations of Motion

In ref.[6] we proposed the following novel p-brane actions:

$$S = -\int d^{p+1}\sigma\, \Phi(\varphi) \Big[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \sqrt{F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}} \Big] \tag{8}$$

with $F_{ab}(A) = \partial_a A_b - \partial_b A_a$, and:

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \ldots i_{p+1}} \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \varphi^{i_1} \ldots \partial_{a_{p+1}} \varphi^{i_{p+1}} \quad , \quad i, j = 1, \ldots, p+1 . \tag{9}$$

Here γ_{ab} and $G_{\mu\nu}$ have the same meaning as in (1).

Let us notice the following significant differences of (8) w.r.t. the standard Nambu-Goto p-branes (in the Polyakov-like formulation) (1):

- New non-Riemannian integration measure density (volume-form) $\Phi(\varphi)$ (9) instead of the usual $\sqrt{-\gamma}$, built entirely in terms of auxiliary world-sheet scalar fields φ^i independent of the Riemannian metric γ_{ab}.
- There is *no* "cosmological-constant" term ($(p-1)\sqrt{-\gamma}$) in (8).
- The action (8) is manifestly Weyl-conformal invariant for *any p*; here Weyl-conformal symmetry is given by Weyl rescaling of γ_{ab} supplemented with a special diffeomorphism in the target space of auxiliary φ-fields:

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\, \gamma_{ab} \quad , \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \;\text{ with }\; \det\left\| \frac{\partial \varphi'^{\,i}}{\partial \varphi^j} \right\| = \rho \; . \tag{10}$$

- There are *no ad hoc* dimensionfull constants in (8); the *variable* brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ is Weyl-conformal *gauge dependent*: $\chi \to \rho^{\frac{1}{2}(1-p)}\chi$.
- The action (8) contains an additional world-volume gauge field A_a in a "square-root" Maxwell (Yang-Mills) Lagrangian[1]; the latter can be straightforwardly generalized to the non-Abelian case: $\sqrt{-\,\mathrm{Tr}\,(F_{ab}(A)F_{cd}(A))\,\gamma^{ac}\gamma^{bd}}$ with $F_{ab}(A) = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$.
- The presence of the world-volume gauge field A_a allows for natural (linear) optional couplings both to external world-volume as well as to space-time "color" charge currents in a Weyl-conformally invariant way (see Eq.(53) below).
- The action (8) describes *intrinsically light-like p*-branes for any odd $(p+1)$ (see Eq.(17) below).

The action (8) yields the following equations of motion w.r.t. auxiliary scalars φ^i:

$$\frac{1}{2}\gamma^{cd}\left(\partial_c X \partial_d X\right) - \sqrt{FF\gamma\gamma} = M \left(= \mathrm{const}\right) , \tag{11}$$

with the short-hand notations:

$$\left(\partial_a X \partial_b X\right) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \quad , \quad \sqrt{FF\gamma\gamma} \equiv \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}} \; . \tag{12}$$

The equations of motion w.r.t. γ^{ab} are:

$$\frac{1}{2}\left(\partial_a X \partial_b X\right) + \frac{F_{ac}\gamma^{cd}F_{db}}{\sqrt{FF\gamma\gamma}} = 0 \; , \tag{13}$$

which upon taking the trace imply $M = 0$ in Eq.(11).

Further we obtain the following equations of motion w.r.t. world-volume gauge field A_a and w.r.t. brane embedding coordinates X^μ, respectively:

$$\partial_b \left(\frac{F_{cd}\gamma^{ac}\gamma^{bd}}{\sqrt{FF\gamma\gamma}} \Phi(\varphi) \right) = 0 \; , \tag{14}$$

$$\partial_a \left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu \right) + \Phi(\varphi)\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \; , \tag{15}$$

where $\Gamma^\mu_{\nu\lambda}$ is the same as in (4).

2.3 Light-Like Branes

Now, let us consider the γ^{ab}-equations of motion (13). Since F_{ab} is an anti-symmetric $(p+1)\times(p+1)$ matrix, it is therefore *not invertible* in any odd $(p+1)$, *i.e.* F_{ab} has at least one zero-eigenvalue vector V^a ($F_{ab}V^b = 0$). Thus, for any odd $(p+1)$ the induced metric:

$$g_{ab} \equiv \left(\partial_a X \partial_b X\right) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \tag{16}$$

[1] "Square-root" Maxwell (Yang-Mills) action in $D = 4$ was originally introduced in [7] and later formulated in dual variables and generalized to "square-root" actions of higher-rank antisymmetric tensor gauge fields in $D \geq 4$ in refs.[8]; see also ref.[9].

on the world-volume of the Weyl-invariant brane (8) is *singular* as *opposed* to the ordinary Nambu-Goto brane where the induced metric is proportional to the intrinsic Riemannian world-volume metric (cf. Eq.(5)). In other words:

$$(\partial_a X \partial_b X)\, V^b = 0 \quad , \quad \text{i.e.}\ (\partial_V X \partial_V X) = 0 \ , \ (\partial_\perp X \partial_V X) = 0 \ , \tag{17}$$

where $\partial_V \equiv V^a \partial_a$ and $\partial_\perp$ are derivates along the tangent vectors in the complement of the tangent vector field V^a.

The constraints (17) imply the following important conclusion: every point on the (fixed-time) world-surface of the Weyl-invariant p-brane (8) (for odd $(p+1)$) moves in orthogonal direction w.r.t. itself with the speed of light in a time-evolution along the zero-eigenvalue vector-field V^a of the world-volume electromagnetic field-strength F_{ab}. Therefore, we will call (8) (for odd $(p+1)$) by the acronym *WILL-brane* (Weyl-Invariant Light-Like-brane) model.

2.4 Dual Formulation of *WILL*-Branes

The A_a-equations of motion (14) can be solved in terms of $(p-2)$-form gauge potentials $\Lambda_{a_1 \ldots a_{p-2}}$ dual w.r.t. A_a. The respective field-strengths are related as follows:

$$F_{ab}(A) = -\frac{1}{\chi} \frac{\sqrt{-\gamma}\, \varepsilon_{abc_1 \ldots c_{p-1}}}{2(p-1)} \gamma^{c_1 d_1} \ldots \gamma^{c_{p-1} d_{p-1}}\, F_{d_1 \ldots d_{p-1}}(\Lambda)\, \gamma^{cd} \left(\partial_c X \partial_d X\right) \ , \tag{18}$$

where:

$$F_{a_1 \ldots a_{p-1}}(\Lambda) = (p-1) \partial_{[a_1} \Lambda_{a_2 \ldots a_{p-1}]} \tag{19}$$

is the $(p-1)$-form dual field-strength, and $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ is the variable brane tension, which we find to be explicitly expressed in terms of the dual field-strength:

$$\chi^2 \equiv \chi^2(\gamma, \Lambda) = -\frac{2}{(p-1)^2}\, \gamma^{a_1 b_1} \ldots \gamma^{a_{p-1} b_{p-1}} F_{a_1 \ldots a_{p-1}}(\Lambda) F_{b_1 \ldots b_{p-1}}(\Lambda) \ . \tag{20}$$

Now, the Biancchi identities for A_a turn into dynamical equations of motion for the dual $(p-2)$-form gauge potentials $\Lambda_{a_1 \ldots a_{p-2}}$:

$$\partial_a \left(\frac{\sqrt{-\gamma}}{\chi(\gamma, \Lambda)} \gamma^{ab} \gamma^{a_1 b_1} \ldots \gamma^{a_{p-2} b_{p-2}} F_{b b_1 \ldots b_{p-2}}(\Lambda)\, \gamma^{cd} \left(\partial_c X \partial_d X\right) \right) = 0 \tag{21}$$

All equations of motion (13),(15) and (21) can be equivalently derived from the following *dual* *WILL*-brane action:

$$S_{\rm dual} = -\frac{1}{2} \int d^{p+1}\sigma\, \chi(\gamma, \Lambda) \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \tag{22}$$

with $\chi(\gamma, \Lambda)$ given in (20) above.

3 The *WILL*-Membrane

The *WILL*-membrane dual action (particular case of (22) for $p=2$) reads:

$$S_{\rm dual} = -\frac{1}{2} \int d^3\sigma\, \chi(\gamma, u) \sqrt{-\gamma} \gamma^{ab} \left(\partial_a X \partial_b X\right) \ , \tag{23}$$

$$\chi(\gamma, u) \equiv \sqrt{-2 \gamma^{cd} \partial_c u \partial_d u} \ , \tag{24}$$

where u is the dual "gauge" potential w.r.t. A_a:

$$F_{ab}(A) = -\frac{1}{2\chi(\gamma, u)} \sqrt{-\gamma} \varepsilon_{abc} \gamma^{cd} \partial_d u\, \gamma^{ef} \left(\partial_e X \partial_f X\right) \ . \tag{25}$$

$S_{\rm dual}$ is manifestly Weyl-invariant (under $\gamma_{ab} \to \rho \gamma_{ab}$).

The equations of motion w.r.t. γ^{ab}, u (or A_a), and X^μ read accordingly:

$$(\partial_a X \partial_b X) + \frac{1}{2}\gamma^{cd}(\partial_c X \partial_d X)\left(\frac{\partial_a u \partial_b u}{\gamma^{ef}\partial_e u \partial_f u} - \gamma_{ab}\right) = 0 \;, \tag{26}$$

$$\partial_a\left(\frac{\sqrt{-\gamma}\gamma^{ab}\partial_b u}{\chi(\gamma,u)}\gamma^{cd}(\partial_c X \partial_d X)\right) = 0 \;, \tag{27}$$

$$\partial_a\left(\chi(\gamma,u)\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi(\gamma,u)\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \;. \tag{28}$$

The first equation above shows that the induced metric $g_{ab} \equiv (\partial_a X \partial_b X)$ has zero-mode eigenvector $V^a = \gamma^{ab}\partial_b u$.

The invariance under world-volume reparametrizations allows to introduce the following standard (synchronous) gauge-fixing conditions:

$$\gamma^{0i} = 0 \;\; (i=1,2) \quad , \quad \gamma^{00} = -1 \;. \tag{29}$$

In spite of the high non-linearity of Eq.(27) for the dual "gauge potential" u, we can easily find solutions by using the following ansatz:

$$u(\tau,\sigma^1,\sigma^2) = \frac{T_0}{\sqrt{2}}\tau \;, \tag{30}$$

where T_0 is an arbitrary integration constant with the dimension of membrane tension. In particular:

$$\chi \equiv \sqrt{-2\gamma^{ab}\partial_a u \partial_b u} = T_0 \tag{31}$$

The ansatz (30) means that we take $\tau \equiv \sigma^0$ to be evolution parameter along the zero-eigenvalue vector-field of the induced metric on the brane ($V^a = \gamma^{ab}\partial_b u = \mathrm{const}\,(1,0,0)$).

With the gauge choice for γ_{ab} (29) the equations of motion w.r.t. γ^{ab} (26) (which are in fact constraints) become (recall $(\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$):

$$(\partial_0 X \partial_0 X) = 0 \quad , \quad (\partial_0 X \partial_i X) = 0 \;, \tag{32}$$

$$(\partial_i X \partial_j X) - \frac{1}{2}\gamma_{ij}\gamma^{kl}(\partial_k X \partial_l X) = 0 \;, \tag{33}$$

Note that Eqs.(33) look exactly like the classical (Virasoro) constraints for an Euclidean string theory with world-sheet parameters (σ^1,σ^2).

The gauge choice for (29) together with the ansatz (30), as well as taking into account (32), bring the the equations of motion w.r.t. u to the form:

$$\partial_0\left(\sqrt{\gamma_{(2)}}\gamma^{kl}(\partial_k X \partial_l X)\right) = 0 \;, \tag{34}$$

where $\gamma_{(2)} = \det\|\gamma_{ij}\|$ $(i,j,k,l = 1,2)$. Eq.(34) is the only remnant from the original A_a-equations of motion (14).

Accordingly, the X^μ-equations of motion now read:

$$\Box^{(3)} X^\mu + \left(-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl}\partial_k X^\nu \partial_l X^\lambda\right)\Gamma^\mu_{\nu\lambda} = 0 \;, \tag{35}$$

where:

$$\Box^{(3)} \equiv -\frac{1}{\sqrt{\gamma^{(2)}}}\partial_0\left(\sqrt{\gamma^{(2)}}\partial_0\right) + \frac{1}{\sqrt{\gamma^{(2)}}}\partial_i\left(\sqrt{\gamma^{(2)}}\gamma^{ij}\partial_j\right) \;. \tag{36}$$

We recall that everywhere in Eqs.(32)–(36) the space-like part of the internal membrane metric γ_{ij} is of the form (42).

4 *WILL*-Memrane Solutions in Non-Trivial Gravitational Backgrounds

4.1 Example: *WILL*-Membrane in Spherically-Symmetric Static Backgrounds

Let us consider a general spherically-symmetric static gravitational background in $D=4$ embedding space-time:

$$(ds)^2 = -A(r)(dt)^2 + B(r)(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2]\;. \tag{37}$$

Specifically we have:

$$A(r) = B^{-1}(r) = 1 - \frac{2GM}{r} \tag{38}$$

for Schwarzschild black hole,

$$A(r) = B^{-1}(r) = 1 - \frac{2GM}{r} + \frac{Q^2}{r^2} \tag{39}$$

for Reissner-Nordström black hole,

$$A(r) = B^{-1}(r) = 1 - \kappa r^2 \tag{40}$$

for (anti-) de Sitter space, *etc.*.

To find solutions of the equations of motion (and constraints) (32)–(36) we will use the following ansatz:

$$X^0 \equiv t = \tau \quad , \quad X^1 \equiv r = r(\tau,\sigma^1,\sigma^2) \quad , \quad X^2 \equiv \theta = \sigma^1 \quad , \quad X^3 \equiv \phi = \sigma^2\;; \tag{41}$$

$$\|\gamma_{ij}\| = a(\tau)\begin{pmatrix} 1 & 0 \\ 0 & \sin^2(\sigma^1)\end{pmatrix} \tag{42}$$

In other words, we assume that the underlying *WILL*-membrane has spherical topology of its fixed-time world-surface.

From Eqs.(32) taking into account (37) we obtain:

$$\frac{\partial}{\partial\tau} r = \pm A(r) \quad , \quad \frac{\partial}{\partial\sigma^i} r = 0\;. \tag{43}$$

From Eq.(34) we get $\frac{\partial}{\partial\tau} r = 0$ which upon combining with (43) gives:

$$r = r_0 \equiv \mathrm{const} \;\;,\;\; \text{where} \quad A(r_0) = 0\;. \tag{44}$$

The X^0-equation of motion (Eq.(35) for $\mu = 0$) implies for the intrinsic *WILL*-membrane metric:

$$\|\gamma_{ij}\| = c_0\, e^{\mp\tau/r_0}\begin{pmatrix} 1 & 0 \\ 0 & \sin^2(\sigma^1)\end{pmatrix}\;, \tag{45}$$

where c_0 is an arbitrary integration constant.

From (44) we conclude that the *WILL*-membrane with spherical topology (and with exponentially blowing-up/deflating radius w.r.t. internal metric) "sits" on (materializes) the event horizon of the pertinent black hole in $D = 4$ embedding space-time.

4.2 Example: *WILL*-membrane in Product-Space Backgrounds

Here we consider *WILL*-membrane moving in a general product-space $D = (d + 2)$-dimensional gravitational background $\mathcal{M}^d \times \Sigma^2$ with coordinates (x^μ, y^m) ($\mu = 0, 1, \ldots, d-1$, $m = 1, 2$) and Riemannian metric $(ds)^2 = f(y)g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n$.

We assume that the *WILL*-brane wraps around the "internal" space Σ^2 and use the following ansatz (recall $\tau \equiv \sigma^0$):

$$X^\mu = X^\mu(\tau) \quad , \quad Y^m = \sigma^m \quad , \quad \gamma_{mn} = a(\tau)\, g_{mn}(\sigma^1,\sigma^2) \tag{46}$$

Then the equations of motion and constraints (32)–(36) reduce to:

$$\partial_\tau X^\mu \partial_\tau X^\nu g_{\mu\nu}(X) = 0 \quad , \quad \frac{1}{a(\tau)}\,\partial_\tau\Big(a(\tau)\partial_\tau X^\mu\Big) + \partial_\tau X^\nu \partial_\tau X^\lambda\, \Gamma^\mu_{\nu\lambda} = 0 \tag{47}$$

where $a(\tau)$ is the conformal factor of the space-like part of the internal membrane metric (last Eq.(46)). Eqs.(47) are of the same form as the equations of motion for a massless point-particle with a world-line "einbein" $e = a^{-1}$ moving in $\mathcal{M}^d$. In other words, the simple solution above describes a membrane living in the extra "internal" dimensions and moving as a whole with the speed of light in "ordinary" space-time.

Notice that although the *WILL*-brane is wrapping the extra dimensions in a topologically non-trivial way (cf. second Eq.(46)), its modes remain *massless* from the projected d-dimensional space-time point of view. This is a highly non-trivial result since we have here particles (membrane modes), which aquire in this way non-zero quantum numbers, while at the same time remaing massless. In contrast, one should recall that in ordinary Kaluza-Klein theory (for a review, see [11]), non-trivial dependence on the extra dimensions is possible for point particles or even standard strings and branes only at a very high energy cost (either by momentum modes or winding modes), which implies a very high mass from the projected $D = 4$ space-time point of view.

4.3 Example: WILL-Membrane in a PP-Wave Background

As a final non-trivial example let us consider *WILL*-membrane dynamics in external plane-polarized gravitational wave (*pp-wave*) background:

$$(ds)^2 = -dx^+dx^- - F(x^+, x^I)\,(dx^+)^2 + dx^I dx^I \;, \tag{48}$$

and employ in (32)–(36) the following natural ansatz for X^μ (here $\sigma^0 \equiv \tau$; $I = 1, \ldots, D-2$) :

$$X^- = \tau \quad , \quad X^+ = X^+(\tau, \sigma^1, \sigma^2) \quad , \quad X^I = X^I(\sigma^1, \sigma^2) \;. \tag{49}$$

The non-zero affine connection symbols for the pp-wave metric (48) are: $\Gamma^-_{++} = \partial_+ F$, $\Gamma^-_{+I} = \partial_I F$, $\Gamma^I_{++} = \frac{1}{2}\partial_I F$.

It is straightforward to show that the solution does not depend on the form of the pp-wave front $F(x^+, x^I)$ and reads:

$$X^+ = X^+_0 = \text{const} \quad , \quad \gamma_{ij} = \tau\text{−independent} \;; \tag{50}$$

$$\partial_i X^I \partial_j X^I - \frac{1}{2}\gamma_{ij}\gamma^{kl}\partial_k X^I \partial_l X^I = 0 \quad , \quad \partial_i \left(\sqrt{\gamma^{(2)}}\gamma^{ij}\partial_j X^I\right) = 0 \tag{51}$$

where the latter equations describe a string embedded in the transverse $(D-2)$-dimensional flat Euclidean space.

5 *WILL*-Membrane as a Source for Gravity and Electromagnetism

In this section we shall consider the Einstein-Maxwell system coupled to an electrically charged *WILL*-membrane, *i.e.*, we shall take into account the back-reaction of the *WILL*-membrane serving as a material and electrically charged source for gravity and electromagnetism. The relevant action reads:

$$S = \int d^4x\,\sqrt{-G}\left[\frac{R}{16\pi G_N} - \frac{1}{4}\mathcal{F}_{\mu\nu}(\mathcal{A})\mathcal{F}_{\kappa\lambda}(\mathcal{A})G^{\mu\kappa}G^{\nu\lambda}\right] + S_{\text{WILL−brane}} \;, \tag{52}$$

where $\mathcal{F}_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, and:

$$S_{\text{WILL−brane}} = -\int d^3\sigma\,\Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}\right] - q\int d^3\sigma\,\varepsilon^{abc}\mathcal{A}_\mu \partial_a X^\mu F_{bc} \;. \tag{53}$$

Note the appearance of a natural Weyl-conformal invariant coupling of the *WILL*-brane to the external space-time electromagnetic field $\mathcal{A}_\mu$ – the last Chern-Simmons-like term in (53). The latter is a special case of a class of Chern-Simmons-like couplings of extended objects to external electromagnetic fields proposed in ref.[10].

The Einstein-Maxwell equations of motion are of the standard form:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi G_N\left(T^{(EM)}_{\mu\nu} + T^{(brane)}_{\mu\nu}\right) \;, \tag{54}$$

$$\partial_\nu \left(\sqrt{-G} G^{\mu\kappa} G^{\nu\lambda} \mathcal{F}_{\kappa\lambda}\right) + j^\mu = 0 \; , \tag{55}$$

where:

$$T^{(EM)}_{\mu\nu} \equiv \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda}G^{\kappa\lambda} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda}G^{\rho\sigma}G^{\kappa\lambda} \; , \tag{56}$$

$$T^{(brane)}_{\mu\nu} \equiv -G_{\mu\kappa}G_{\nu\lambda}\int d^3\sigma \, \frac{\delta^{(4)}\Big(x - X(\sigma)\Big)}{\sqrt{-G}} \, \Phi(\varphi)\gamma^{ab}\partial_a X^\kappa \partial_b X^\lambda \; , \tag{57}$$

$$j^\mu \equiv q \int d^3\sigma \, \delta^{(4)}\Big(x - X(\sigma)\Big)\varepsilon^{abc}F_{bc}\partial_a X^\mu \; . \tag{58}$$

For the *WILL*-membrane subsystem we can use instead of the action (53) its dual one (similar to the simpler case Eq.(8) versus Eq.(23)):

$$S^{\rm dual}_{\rm WILL-brane} = -\frac{1}{2}\int d^3\sigma \, \chi(\gamma, u, \mathcal{A}) \sqrt{-\gamma}\gamma^{ab}\left(\partial_a X \partial_b X\right) \; , \tag{59}$$

where the variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ is given by:

$$\chi(\gamma, u, \mathcal{A}) \equiv \sqrt{-2\gamma^{cd}\left(\partial_c u - q\mathcal{A}_c\right)\left(\partial_d u - q\mathcal{A}_d\right)} \quad , \quad \mathcal{A}_a \equiv \mathcal{A}_\mu \partial_a X^\mu \; . \tag{60}$$

Here u is the dual "gauge" potential w.r.t. A_a and the corresponding field-strength and dual field-strength are related as (cf. Eq.(25)) :

$$F_{ab}(A) = -\frac{1}{2\chi(\gamma, u, \mathcal{A})}\sqrt{-\gamma}\varepsilon_{abc}\gamma^{cd}\left(\partial_d u - q\mathcal{A}_d\right)\gamma^{ef}\left(\partial_e X \partial_f X\right) \; . \tag{61}$$

The corresponding equations of motion w.r.t. γ^{ab}, u (or A_a), and X^μ read accordingly:

$$\left(\partial_a X \partial_b X\right) + \frac{1}{2}\gamma^{cd}\left(\partial_c X \partial_d X\right)\left(\frac{\left(\partial_a u - q\mathcal{A}_a\right)\left(\partial_b u - q\mathcal{A}_b\right)}{\gamma^{ef}\left(\partial_e u - q\mathcal{A}_e\right)\left(\partial_f u - q\mathcal{A}_f\right)} - \gamma_{ab}\right) = 0 \; ; \tag{62}$$

$$\partial_a\left(\frac{\sqrt{-\gamma}\gamma^{ab}\left(\partial_b u - q\mathcal{A}_b\right)}{\chi(\gamma, u, \mathcal{A})}\gamma^{cd}\left(\partial_c X \partial_d X\right)\right) = 0 \; ; \tag{63}$$

$$\partial_a\left(\chi(\gamma, u, \mathcal{A})\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi(\gamma, u, \mathcal{A})\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} - q\varepsilon^{abc}F_{bc}\partial_a X^\nu\left(\partial_\lambda \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\lambda\right)G^{\lambda\mu} = 0 \; . \tag{64}$$

Following steps similar to the ones in the previous section we obtain the following self-consistent spherically symmetric stationary solution for the full coupled Einstein-Maxwell-*WILL*-membrane system (52). For the Einstein subsystem we have a solution:

$$(ds)^2 = -A(r)(dt)^2 + A^{-1}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2] \; , \tag{65}$$

consisting of two different black holes with a *common* event horizon:

- Schwarzschild black hole inside the horizon:

$$A(r) \equiv A_-(r) = 1 - \frac{2GM_1}{r} \quad , \quad \text{for } \; r < r_0 \equiv r_{\rm horizon} = 2GM_1 \; . \tag{66}$$

- Reissner-Norström black hole outside the horizon:

$$A(r) \equiv A_+(r) = 1 - \frac{2GM_2}{r} + \frac{GQ^2}{r^2} \quad , \quad \text{for } \; r > r_0 \equiv r_{\rm horizon} \; , \tag{67}$$

  where $Q^2 = 8\pi q^2 r^4_{\rm horizon} \equiv 128\pi q^2 G^4 M_1^4$;

For the Maxwell subsystem we have $\mathcal{A}_1 = \ldots = \mathcal{A}_{D-1} = 0$ everywhere and:

- Coulomb field outside horizon:

$$\mathcal{A}_0 = \frac{\sqrt{2}\, q\, r^2_{\rm horizon}}{r} \quad , \quad \text{for} \;\; r \geq r_0 \equiv r_{\rm horizon} \; . \tag{68}$$

- No electric field inside horizon:

$$\mathcal{A}_0 = \sqrt{2}\, q\, r_{\rm horizon} = \text{const} \quad , \quad \text{for} \;\; r \leq r_0 \equiv r_{\rm horizon} \; . \tag{69}$$

For the *WILL*-membrane subsystem the corresponding solution reads:

$$X^0 \equiv t = \tau \quad , \quad \theta = \sigma^1 \quad , \quad \phi = \sigma^2 \quad , \quad r(\tau, \sigma^1, \sigma^2) = r_{\rm horizon} = \text{const} \; , \tag{70}$$

where $A_\pm(r_{\rm horizon}) = 0$, *i.e.*, the *WILL*-membrane "sits" on (materializes) the common event horizon of the pertinent black holes. Furthermore, the presence of the *WILL*-membrane entails an important matching condition for the metric components along its surface[2] :

$$\frac{\partial}{\partial r} A_+ \Big|_{r=r_{\rm horizon}} - \frac{\partial}{\partial r} A_- \Big|_{r=r_{\rm horizon}} = -16\pi G \chi \; , \tag{71}$$

which yields the following relations between the parameters of the black holes and the *WILL*-membrane (q being its surface charge density) :

$$M_2 = M_1 + 32\pi q^2 G^3 M_1^3 \tag{72}$$

and for the brane tension χ:

$$\chi \equiv T_0 - 2q^2 r_{\rm horizon} = q^2 G M_1 \quad , \;\; \text{i.e.} \;\; T_0 = 5q^2 G M_1 \; . \tag{73}$$

Let us stress that the present *WILL*-brane models provide a systematic description of light-like branes from first principles starting with concise Weyl-conformal invariant actions (8), (52)–(53). As a consequence, these actions also yield additional information impossible to obtain within the phenomenological approach to light-like thin shell dynamics [12] (*i.e.*, where the membranes are introduced *ad hoc*), such as the requirement that the light-like brane must sit on the (common) event horizon(s) of the pertinent black hole(s).

6 Conclusions and Outlook

In the present work we have discussed a novel class of Weyl-invariant p-brane theories whose dynamics significantly differs from ordinary Nambu-Goto p-brane dynamics. The princial ingredients of our construction are:

- Alternative non-Riemannian integration measure (volume-form) (9) on the p-brane world-volume independent of the intrinsic Riemannian metric;

- Acceptable dynamics in the novel class of brane models (Eqs.(8),(53)) *naturally* requires the introduction of additional world-volume gauge fields.

- By employing square-root Yang-Mills actions for the pertinent world-volume gauge fields one achieves manifest *Weyl-conformal symmetry* in the new class of p-brane theories *for any* p.

- The brane tension is *not* a constant dimensionful scale given *ad hoc*, but rather it appears as a *composite* world-volume scalar field (Eqs.(20),(24),(60)) transforming non-trivially under Weyl-conformal transformations.

- The novel class of Weyl-invariant p-brane theories describes intrinsically *light-like* p-branes for any even p (*WILL*-branes).

[2] The matching condition (71) corresponds to the statically soldering conditions in the phenomenological theory of light-like thin shell dynamics in general relativity [12].

- When put in a gravitational black hole background, the *WILL*-membrane ($p = 2$) sits on ("materializes") the event horizon.

- When moving in background product-spaces ("Kaluza-Klein" context) the *WILL*-membrane describes *massless* modes, even though the membrane is wrapping the extra dimensions and therefore aquiring non-trivial Kaluza-Klein charges.

- The coupled Einstein-Maxwell-*WILL*-membrane system (52) possesses self-consistent solution where the *WILL*-membrane serves as a material and electrically charged source for gravity and electromagnetism, and it "sits" on (materializes) the common event horizon for a Schwarzschild (in the interior) and Reissner-Nordström (in the exterior) black holes. Thus our model (52) provides an explicit dynamical realization of the so called "membrane paradigm" in the physics of black holes [13].

- The *WILL*-branes could be good representations for the string-like objects introduced by 't Hooft in ref.[14] to describe gravitational interactions associated with black hole formation and evaporation, since as shown above the *WILL*-branes locate themselves automatically in the horizons and, therefore, they could represent degrees of freedom associated particularly with horizons.

The novel class of Weyl-conformal invariant p-branes discussed above suggests various physically interesting directions for further study: quantization (Weyl-conformal anomaly and critical dimensions); supersymmetric generalization; possible relevance for the open string dynamics (similar to the role played by Dirichlet- (Dp-)branes); *WILL*-brane dynamics in more complicated gravitational black hole backgrounds (*e.g.*, Kerr-Newman).

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